$$\ln L(\mu,\sigma^2) = -\frac{N}{2}\ln 2\pi - \frac{N}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{N}(y_i-\mu)^2$$

$$\begin{bmatrix} \frac{\partial \ln L}{\partial \mu} \\ \frac{\partial \ln L}{\partial \sigma^2} \end{bmatrix}$$

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{N}(y_i-\mu), \quad \frac{\partial \ln L}{\partial \sigma^2} = -\frac{N}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{N}(y_i-\mu)^2$$

Information Matrix

$$\frac{\partial^2 \ln L}{\partial \mu^2} = -\frac{N}{\sigma^2}, \quad \frac{\partial^2 \ln L}{\partial (\sigma^2)^2} = \frac{N}{2\sigma^4} - \frac{1}{\sigma^6}\sum_{i=1}^{N}(y_i-\mu)^2$$

常数

$$\frac{\partial^2 \ln L}{\partial \mu \partial \sigma^2} = -\frac{1}{\sigma^4}\sum_{i=1}^{N}(y_i-\mu)$$

$$E\left[\frac{\partial^2 \ln L}{\partial (\sigma^2)^2}\right] = \frac{N}{2\sigma^4} - \frac{N}{\sigma^4} = -\frac{N}{2\sigma^4}$$

$$-E_{\theta_0}\left[\frac{\partial^2 \ln L}{\partial \theta \partial \theta'}\bigg|_{\theta_0}\right] = \begin{bmatrix} \frac{N}{\sigma^2} & 0 \\ 0 & \frac{N}{2\sigma^4} \end{bmatrix}$$

① 期望值法：把 $\hat{\sigma}^2$ 代入

② OIM：直接用 y_i, $\hat{\sigma}^2$, $\hat{\mu}$ 代入

③ BHHH：用 $\sum_{i=1}^{N} \hat{s}_i \hat{s}_i'$