$$y = \beta_0 + \beta_1 x_1 + \cdots + \beta_K x_K + \gamma q + v$$

$$E[v \mid x_1, \cdots x_K, q] = 0.$$

OLS:

$$b = (x'x)^{-1}x'y$$
$$= (x'x)^{-1}x'(x\beta + \gamma q + v)$$
$$\to \beta + \gamma E[\vec{x}\vec{x}']^{-1} E[\vec{x}q]$$

$$\beta + \gamma \cdot \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 \\ E(x_2 q) \end{bmatrix}$$

感兴趣: β_1. 和 q 没关系.

$$\hat{\beta}_1 \to \beta_1 + \gamma \cdot A_{12} E(x_2 q)$$

控制变量: β_K. 和 q 有关系.

$$E[\vec{x}'q] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ E[x_K q] \end{bmatrix}$$