

如果 proxy z 和 q 相关的.

$$y = \beta_0 + \beta_1 x + u$$

$$E[u] = 0, \quad \text{cov}(z, u) = 0$$

$\tilde{\beta}$ 是 OLS, y 对 $1, x$ 回归.

$$p\lim(\tilde{\beta}) = \beta + \frac{\text{cov}(x, u)}{\text{Var}(x)} \quad \beta + E(x'x)^{-1} E[xu]$$

$x = [1, x_1]'$

$\hat{\beta}$ 是加上 z 回归.

$$p\lim(\hat{\beta}) = \beta + \frac{\text{cov}(a, u)}{\text{Var}(a)} \quad \leftarrow$$

a 是 x 对 $1, z$ 回归的残差.

$$a = x - \pi_0 - \pi_1 z$$

比较 $\frac{|\text{cov}(x, u)|}{\text{Var}(x)}$ 和 $\frac{|\text{cov}(a, u)|}{\text{Var}(a)}$

$$" \text{cov}(z, u) = 0 ",$$

$$\begin{aligned} \text{cov}(a, u) &= \text{cov}(x - \pi_0 - \pi_1 z, u) \\ &= \text{cov}(x, u) \end{aligned}$$

$$\text{Var}(x) = \text{Var}(\pi_0 + \pi_1 z + a)$$

$$= \pi_1^2 \text{Var}(z) + \text{Var}(a)$$