

Consistency

$$b = (X'X)^{-1}X'y = \beta + (X'X)^{-1}X'\varepsilon$$

$$\left(\frac{X'X}{N}\right)^{-1} \left(\frac{X'\varepsilon}{N}\right) \rightarrow \text{plim} \left(\frac{X'X}{N}\right)^{-1} \text{plim} \left(\frac{X'\varepsilon}{N}\right)$$

$$\rightarrow \left(\text{plim} \frac{X'X}{N}\right)^{-1} \text{plim} \left(\frac{X'\varepsilon}{N}\right)$$

$$\rightarrow A^{-1} \underbrace{E(\vec{x}_i \varepsilon_i)}_{=0, \text{Cont. Exo.}} = \vec{0}$$

Asymptotic Distribution

$b \rightarrow$ 常数 β

$$\sqrt{N}(b - \beta)$$

CLT: $\sqrt{N}(\hat{\theta}_N - \theta) \stackrel{d}{\rightarrow} \text{Normal}$, $\hat{\theta}_N$ 样本均值.

小样本没有CLT. 假设 Normal.

大样本. $\vec{g}_i \equiv \vec{x}_i \varepsilon_i$, $\{\vec{g}_i\}$ ergodic stationary, M.D.S.

$$\sqrt{N}\bar{g} \rightarrow N(\vec{0}, E[\vec{g}_i \vec{g}_i']), \quad \bar{g} \equiv \frac{1}{N} \sum_{i=1}^N \vec{g}_i$$

$$\sqrt{N}(b - \beta) = \sqrt{N} \left(\frac{X'X}{N}\right)^{-1} \left(\frac{X'\varepsilon}{N}\right)$$

$$= \left(\frac{1}{N} \sum_{i=1}^N \vec{x}_i \vec{x}_i'\right)^{-1} \sqrt{N} \left(\frac{1}{N} \sum_{i=1}^N \vec{x}_i \varepsilon_i\right)$$

$$\rightarrow N(0, \Sigma)$$

$$\Sigma = E[\vec{x}_i \vec{x}_i']^{-1} E[\vec{g}_i \vec{g}_i'] E[\vec{x}_i \vec{x}_i']^{-1}$$