

无偏性

$$E[\hat{\beta}_{IV}] = \beta + E[(Z'X)^{-1}Z'u]$$

$$E[u|Z] = 0.$$

$$E[(Z'X)^{-1}Z'E[u|Z]] \quad ?$$

$$E[u|Z, X] = 0 \quad \text{没意义.}$$

Consistency

$$y = \beta_0 + \beta_1 x_1 + u, \quad \hat{\beta}_1 \rightarrow \beta_1 + \frac{\text{Cov}(Z_1, u)}{\text{Cov}(Z_1, x_1)}$$

$$= \beta_1 + \frac{\sigma_{Z_1} \cdot \sigma_u \cdot \text{Corr}(Z_1, u)}{\sigma_{Z_1} \cdot \sigma_{x_1} \cdot \text{Corr}(Z_1, x_1)}$$

$\downarrow \neq 0$   
 $\text{Corr}(Z_1, x_1) \downarrow, \text{误差大.}$

OLS

$$\hat{\beta}_1 \rightarrow \beta_1 + \frac{\text{Cov}(x_1, u)}{\text{Var}(x_1)} = \beta_1 + \frac{\sigma_u}{\sigma_{x_1}} \cdot \text{Corr}(x_1, u)$$

$$\frac{\sigma_u}{\sigma_{x_1}} |\text{Corr}(Z_1, u)| > |\text{Corr}(x_1, u)| \quad \underline{\text{Corr}(Z_1, x_1)}$$