

$$\Sigma = \begin{bmatrix} \sigma_u^2 + \sigma_\varepsilon^2 & \sigma_u^2 & \dots & \sigma_u^2 \\ \sigma_u^2 & \sigma_u^2 + \sigma_\varepsilon^2 & \dots & \sigma_u^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_u^2 & \sigma_u^2 & \dots & \sigma_u^2 + \sigma_\varepsilon^2 \end{bmatrix}$$

$$= \sigma_\varepsilon^2 I_T + \sigma_u^2 j_T j_T'$$

$$= \sigma_\varepsilon^2 I_T + T \sigma_u^2 j_T (j_T' j_T)^{-1} j_T'$$

$$\Sigma^{-\frac{1}{2}} = \frac{1}{\sigma_\varepsilon} \left(I_T - \theta \cdot j_T (j_T' j_T)^{-1} j_T' \right)$$

$$\theta = 1 - \left(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + T \sigma_u^2} \right)^{\frac{1}{2}} \quad \equiv C_T = I_T - \theta P_T$$

$$\Omega^{-1} = C' C$$

$$C_T y_i = C_T x_i \beta + C_T v_i \quad (GLS)$$

$$y_{it} - \theta \bar{y}_i = (x_{it} - \theta \bar{x}_i) \beta + v_{it} - \theta \bar{v}_i$$

Summary.

FE vs. RE : FE - 致, RE 更有效

FE vs. FD : 都一致, $\{\varepsilon_{it}\}$ iid. 时 FE 更有效.