$$E[x_i x_i']^{-1} E[x_i y_i]$$

$$Q_T \equiv I_T - j_T (j_T' j_T)^{-1} j_T' , \quad j_T = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{T\times 1}$$

$$Q_T = \begin{bmatrix} 1-\frac{1}{T} & -\frac{1}{T} & \cdots & -\frac{1}{T} \\ -\frac{1}{T} & 1-\frac{1}{T} & \cdots & -\frac{1}{T} \\ \vdots & & \ddots & \\ -\frac{1}{T} & -\frac{1}{T} & \cdots & 1-\frac{1}{T} \end{bmatrix} ,$$

$$Q_T \begin{bmatrix} x_{i1} \\ \vdots \\ x_{iT} \end{bmatrix} = \begin{bmatrix} x_{i1} - \bar{x}_i \\ \vdots \\ x_{iT} - \bar{x}_i \end{bmatrix}$$

$$\sqrt{N}(\hat{\beta}_{FE} - \beta) = \left(N^{-1} \sum_{i=1}^{N} \tilde{x}_i' \tilde{x}_i\right)^{-1} \left(N^{-\frac{1}{2}} \sum_{i=1}^{N} \tilde{x}_i' \tilde{\varepsilon}_i\right)$$

$$\tilde{x}_i' \tilde{\varepsilon}_i = x_i' Q' Q \varepsilon_i = \tilde{x}_i' \varepsilon_i$$

假设 $E[\varepsilon_i \varepsilon_i' \mid x_i, u_i] = E[\varepsilon_i \varepsilon_i'] = \sigma_\varepsilon^2 I_T$

$$\widehat{Avar}(\hat{\beta}_{FE}) = \hat{\sigma}_\varepsilon^2 \left(\sum_{i=1}^{N} \tilde{x}_i' \tilde{x}_i\right)^{-1}$$