

$$\underline{F_n \rightarrow F}$$

$$E[1(x_i \leq x)] = 1 \cdot P(x_i \leq x) + 0 \cdot (1 - P(x_i \leq x))$$

$$= p$$

$$Var(1(x_i \leq x)) = p \cdot \overset{\substack{\text{均值} \\ \downarrow}}{1-p}^2 + (1-p)(0-p)^2$$

$$= p(1-p)$$

$$E\left[\frac{1}{n} \sum_{i=1}^n 1(x_i \leq x)\right] = \frac{1}{n} \cdot n \cdot p = F(x)$$

$$Var\left(\frac{1}{n} \sum_{i=1}^n 1(x_i \leq x)\right) = \frac{1}{n^2} n \cdot p(1-p) \xrightarrow{n \rightarrow \infty} 0$$

$$F_n \xrightarrow{m.s.} F \Rightarrow F_n \xrightarrow{P} F$$