

Rubin's Causal Model

$$\text{potential outcome} = \begin{cases} y_{1i}, & \text{if } X_i = 1 \\ y_{0i}, & \text{if } X_i = 0 \end{cases}$$

$$\begin{aligned} \text{observed outcome } y_i &= \begin{cases} y_{1i}, & \text{if } x_i = 1 \\ y_{0i}, & \text{if } x_i = 0 \end{cases} \\ &= y_{0i} + \underbrace{(y_{1i} - y_{0i})}_{\text{Causal effect}} x_i \end{aligned}$$

Naive Comparison

$$\begin{aligned} E[y_i | x_i = 1] - E[y_i | x_i = 0] &= E[y_{1i} | x_i = 1] - E[y_{0i} | x_i = 1] \quad \text{ATET} \\ &\quad + \underbrace{E[y_{0i} | x_i = 1] - E[y_{0i} | x_i = 0]}_{\text{Selection Bias}} \end{aligned}$$

Random Assignment

$$\begin{aligned} E[y_i | x_i = 1] - E[y_i | x_i = 0] &= E[y_{1i} | x_i = 1] - E[y_{0i} | x_i = 0] \\ &= E[y_{1i} | x_i = 1] - E[y_{0i} | x_i = 1] \\ &= E[y_{1i} - y_{0i} | x_i = 1] = E[y_{1i} - y_{0i} | x_i = 0] \\ &= E[y_{1i} - y_{0i}] \end{aligned}$$