Efficient GMM

$\hat{W} = \hat{S}^{-1}$ 时，证明

$$(\Sigma_{zx}' W \Sigma_{zx})^{-1} \Sigma_{zx}' W \overset{S^{-1}}{\downarrow} S W \Sigma_{zx} (\Sigma_{zx}' W \Sigma_{zx})^{-1}$$

$$\geq (\Sigma_{zx}' S^{-1} \Sigma_{zx})^{-1} \qquad (\Sigma_{zx}' \ \ )^{-1} \quad L, K.$$

证：$A \geq B$，$A - B$ p.s.d.，$B^{-1} - A^{-1}$ p.s.d.

$\exists\, C'C = S^{-1}$ (因 S p.d.)

$$S = (C'C)^{-1} = C^{-1}(C^{-1})'$$

$$B^{-1} - A^{-1} = \Sigma_{zx}' C'C \Sigma_{zx}$$

$$- \Sigma_{zx}' W \Sigma_{zx} (\Sigma_{zx}' W C^{-1}(C^{-1})' W \Sigma_{zx})^{-1} \Sigma_{zx} W \Sigma_{zx}$$

$$= H'H - H'\underline{(C^{-1})' W \Sigma_{zx}} (\Sigma_{zx}' W C^{-1}(C^{-1})' W \Sigma_{zx})^{-1}$$

$$\Sigma_{zx}' W C^{-1} H$$

定义 $G \equiv (C^{-1})' W \Sigma_{zx}$

$$= H'H - H'G(G'G)^{-1}G'H$$

$$= H'(I_L - G(G'G)^{-1}G')H = H'M_G' M_G H \geq 0$$