

# NW Estimator

$$b - \beta = (X'X)^{-1} X' \varepsilon = \left( \frac{1}{n} X'X \right)^{-1} \underbrace{\left( \frac{1}{n} X' \varepsilon \right)}_{\equiv \bar{g}}$$

$$\sqrt{n}(b - \beta) ? \quad \text{Var}(\sqrt{n} \cdot \frac{1}{n} X' \varepsilon) ?$$

Gordin:  $\sqrt{n}(\bar{g}) \xrightarrow{d} N(0, \lim_{n \rightarrow \infty} \text{Var}(\sqrt{n} \bar{g}))$

$\vec{g}_t$  有  $\vec{g}_t$  序列 相关.  $E[g_t g_{t-j}] \neq 0$ .

$k \times 1$   $\text{Var}(g_t) = \Gamma_0$

$\text{Cov}(g_t, g_{t+j}) = \Gamma_j$

$\text{Cov}(g_{t+j}, g_t) = \Gamma_{-j}$ ,  $\Gamma_j = \Gamma_{-j}'$

$$\lim_{n \rightarrow \infty} \text{Var}(\sqrt{n} \bar{g}) = \frac{1}{n} \text{Var}\left(\sum_{i=1}^n g_i\right) = \frac{1}{n} \text{Var}(g_1 + \dots + g_n)$$

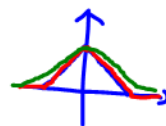
$$= \frac{1}{n} \left( n\Gamma_0 + (n-1)\Gamma_1 + (n-2)\Gamma_2 + \dots + \Gamma_{n-1} + (n-1)\Gamma_{-1} + (n-2)\Gamma_{-2} + \dots + \Gamma_{-(n-1)} \right)$$

$$\xrightarrow{n \rightarrow \infty} = \sum_{j=-\infty}^{\infty} \Gamma_j = \Gamma_0 + \sum_{j=1}^{\infty} \Gamma_j + \Gamma_j'$$

若  $p$ .  $\hat{\Gamma}_0 + \sum_{j=1}^p \hat{\Gamma}_j + \hat{\Gamma}_j'$  (Hansen - White (1982)). 不正定.

kernel-based estimator

$$\hat{S} = \sum_{j=-p+1}^{p-1} k\left(\frac{j}{p}\right) \hat{\Gamma}_j$$



Barlett kernel.

Newey - West

$$k(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

例:  $p=3$ .  $\hat{S} = \sum_{j=-3+1}^{3-1} k\left(\frac{j}{3}\right) \hat{\Gamma}_j = k\left(-\frac{2}{3}\right) \hat{\Gamma}_{-2} + k\left(-\frac{1}{3}\right) \hat{\Gamma}_{-1} + k(0) \hat{\Gamma}_0 + k\left(\frac{1}{3}\right) \hat{\Gamma}_1 + k\left(\frac{2}{3}\right) \hat{\Gamma}_2$