

Entropy

Two states, A or B
50% 50%

$$0.5 \rightarrow 1 \quad \log_2 \frac{1}{0.5} = 1 \text{ bit.}$$

Reduce the uncertainty by a factor of 2.

A: 75% , B: 25%

$$\text{B is } \frac{1}{4} \text{ } \log_2 \frac{1}{0.25} = 2$$

$$\text{A is } \frac{3}{4} \text{ } \log_2 \frac{1}{0.75} \approx 0.41$$

$$0.75 \times 0.41 + 0.25 \times 2$$

$$\text{Entropy: } - \sum_i p_i \log p_i$$

Cross-Entropy

A	B	C	D
0.25	0.25	0.25	0.25
00	01	10	11

$$(-0.25 \log 0.25) \times 4 = 2 \text{ bit.}$$

$$(2 \times 0.25) \times 4 = 2 \text{ bit.}$$

0.5 0.25 0.125 0.125

$$-(0.5 \log 0.5 + 0.25 \log 0.25 + \dots)$$

$$= 1.75$$

0 10 110 111

$$0.5 \times 1 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3$$

$$= 1.75$$

0 1 10 11 ?