

## Entropy

Two states, A or B  
50%      50%

$$0.5 \rightarrow 1. \quad \log_2 \frac{1}{0.5} = 1 \text{ bit.}$$

Reduce the uncertainty by a factor of 2.

A: 75%, B: 25%

$$\text{Shift B by 3.} \quad \log_2 \frac{1}{0.25} = 2$$

$$A \neq 3. \quad \log_2 \frac{1}{0.75} \approx 0.41$$

$$0.75 \times 0.41 + 0.25 \times 2 \\ \text{Entropy: } - \sum_i p_i \log p_i$$

## Cross-Entropy

$$\begin{array}{cccc} A & B & C & D \\ 0.25 & 0.25 & 0.25 & 0.25 \end{array} \quad (-0.25 \log 0.25) \times 4 = 2 \text{ bit.}$$

$$\begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \quad (2 \times 0.25) \times 4 = 2 \text{ bit.}$$

$$0.5 \quad 0.25 \quad 0.125 \quad 0.125 \\ - (0.5 \log 0.5 + 0.25 \log 0.25 + \dots) \\ = 1.75$$

$$\begin{array}{cccc} 0 & 10 & 110 & 111 \\ 0.5 \times 1 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3 \\ = 1.75 \end{array}$$

$$\begin{array}{cccc} 0 & 1 & 10 & 11 & ? \end{array}$$